N92-22065

LIMITATIONS ON SQUEEZING AND FORMATION OF THE SUPERPOSITION OF TWO MACROSCOPICALLY DISTINGUISHABLE STATES AT FUNDAMENTAL FREQUENCY IN THE PROCESS OF SECOND HARMONIC GENERATION.

Nikitin S.P., Masalov A.V.
Lebedev Physical Institute

Moscow, USSR

In this paper the results of numerical simulations of quantum state evolution in the process of second harmonic generation (SHG) are discussed. It is shown that at a particular moment of time in the fundamental mode initially coherent state turns into a superposition of two macroscopically distinguishable states. The question if this superposition exhibits quantum interference is analyzed.

To describe the SHG we use the following Hamiltonian:

$$H = \hbar\omega\, a^{\dagger}a + 2\hbar\omega b^{\dagger}b + g\hbar(a^{\dagger}a^{\dagger}b + aab^{\dagger})$$

Here a, $a^{\dagger}$, b, $b^{\dagger}$ are annihilation and creation operators of the fundamental mode and harmonic mode respectively, and g is a coupling constant proportional to the nonlinearity of the medium. The nonlinear interaction is described by the last term in the Hamiltonian. This Hamiltonian corresponds to the case when there is no absorption loss in the medium. The initial quantum state was taken to be a coherent state in the fundamental mode and vacuum state in the harmonic mode.

In our calculations we have used a number-state basis in which a quantum state is just a vector and operators are matrices of c-numbers. Details of our calculations are described in Ref. 1. Earlier similar calculations have been made by Walls and Barakat. It is known that squeezing in the SHG has a minimum. It is shown in Ref. 1 that this minimum appears due to the formation at the fundamental frequency of the superposition of macroscopically distinguishable states. It is the formation of this superposition that is the limiting factor of the largest squeezing achievable in the process.

Fig.1 represents the dependence of amplitude squeezing in the fundamental mode versus the dimensionless time $\tau=gt\sqrt{2N}$. N is the initial average number of photons in the fundamental mode. Fig 2 represents the quasiprobability distribution for the fundamental mode $Q(\alpha)=\langle\alpha|\rho|\alpha\rangle/\pi$ when this superposition is formed. Here ρ is the density matrix of the quantum state and $|\alpha\rangle$ is a coherent state described by a c-number α. Earlier, in Ref.2 it was shown that superposition of two coherent states can be obtained using Kerr nonlinearity. The SHG process appears to be alternative nonlinear process in which the superposition can be obtained.

The question of the origin of this superposition is discussed in Ref. 1 where this phenomenon is attributed to the instability of the SHG process with respect to the initial harmonic phase which is completely uncertain for the initial vacuum state in the harmonic mode. This instability was illustrated by a classical equation solution where quantum uncertainty of the harmonic state and the fundamental state was imitated by randomized initial conditions distributed by the normal law with the same dispersion as quantum states.

Here we would like to pay more attention to the question of whether the superposition is coherent, that is, a pure quantum state, or whether it is a statistical mixture of two coherent states. In order to answer this question one usually uses simple numerical criteria such as $T = \mathrm{Tr}\rho^2$ For a pure state $T = 1$ while for a statistical mixture $T < 1$. The dependence of T versus τ is shown on the Fig. 3. If N=10 the superposition appears at $\tau = 4$. It is clearly seen on Fig. 3 that T at this time is very far from parameter specific to the pure state. So, one can expect that no quantum interference effects could be seen in this state. However, we may check it directly using the density matrix.

To see quantum interference we may consider the function $P(x)=\langle x|\rho|x\rangle$. Here $|x\rangle$ is an eigenstate of a quadrature operator $x = (a+a^{+})/\sqrt{2}$. Experimentally this function P(x) can be obtained using homodyne measurements. It is known that for a coherent state this function is a gaussian. If we calculate this function for a statistical mixture of two coherent states then we get the sum of two gaussians and no quantum interference. For a quantum superposition of two macroscopically distinguishable state this function exhibits an interference pattern. It is therefore interesting to check if the superposition formed in the process of the SHG exhibits quantum interference pattern in P(x).

Fig. 4 represents P(x) calculated from the density matrix of the superposition at $\tau=4$ and N=10. This function obviously exhibits quantum interference, though visibility of the interference pattern is less than for a pure superposition of two coherent states. This result could be explained if we assume that the main portion of the statistical mixture, which in fact the above-mentioned superposition is, is a quantum superposition of two coherent states. Other states which the mixture contains reduce visibility of the interference but can not destroy it completely. Thus the superposition formed in the process of the SHG can exhibit quantum interference though, generally speaking this superposition is a statistical mixture rather than a pure state.

Conclusions

Squeezing in the process of the SHG is limited because of the formation of the superposition of macroscopically distinguishable states at the fundamental frequency. This superposition forms because of the quantum phase uncertainty of the initial harmonic state. Though this superposition is not a pure quantum state, it does exhibit quantum interference in P(x). This fact illustrates that analysis of simple numerical criteria such as $\mathrm{Tr}\,\rho^2$ is not enough to decide whether quantum interference appears or not.

REFERENCES:

1 Nikitin S.P., Masalov A.V., 1991 accepted for publication in Quantum Optics.

2. Yurke B. and Stoler D., 1986, "Generating Quantum Mechanical Superpositions of Macroscopically Distinguishable States via Amplitude Dispersion," Phys. Rev. Lett., 57(1), pp.13-16

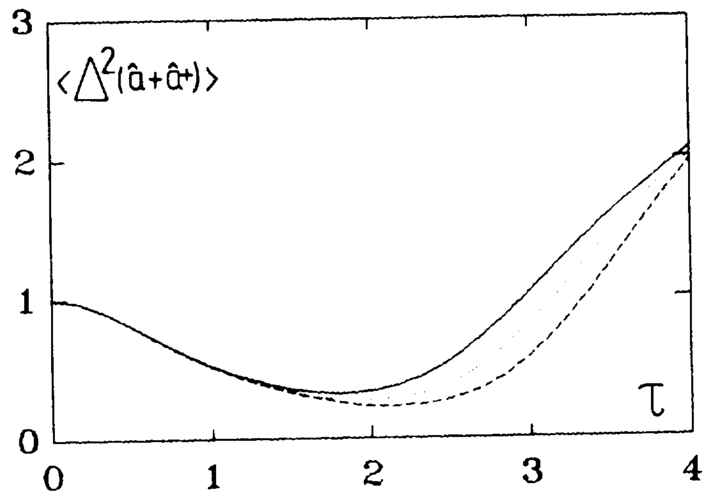

Fig. 1 Squuezing in the fundamental mode vs $\tau=gt\sqrt{2N}$. N=10 (solid line) N=20 (dotted line), N=40 (dashed line)

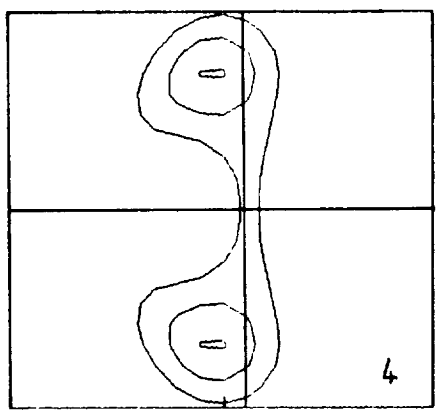

Fig. 2
Quasiprobability $Q(\alpha) = \langle\alpha|\rho|\alpha\rangle/\pi$ for fundamental mode at $\tau=4$; N= 10. Contours at $0.1/\pi$, $0.2/\pi$, $0.3/\pi$.

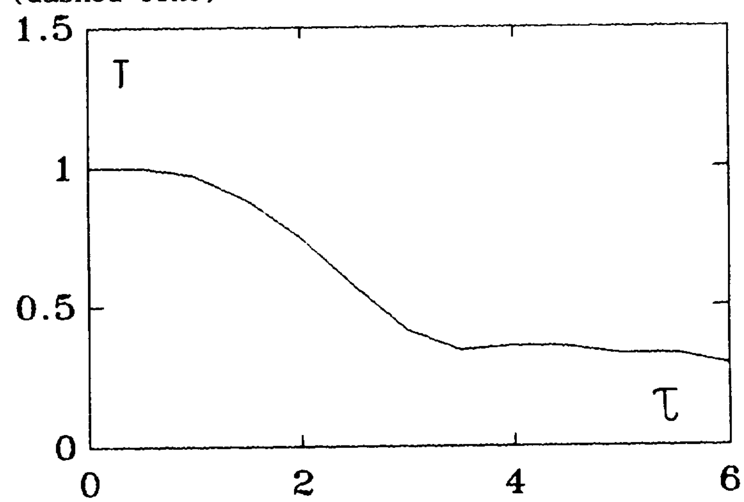

Fig. 3
$T = Tr\rho^2$ versus τ for the fundamental mode. Average photon number N = 10.

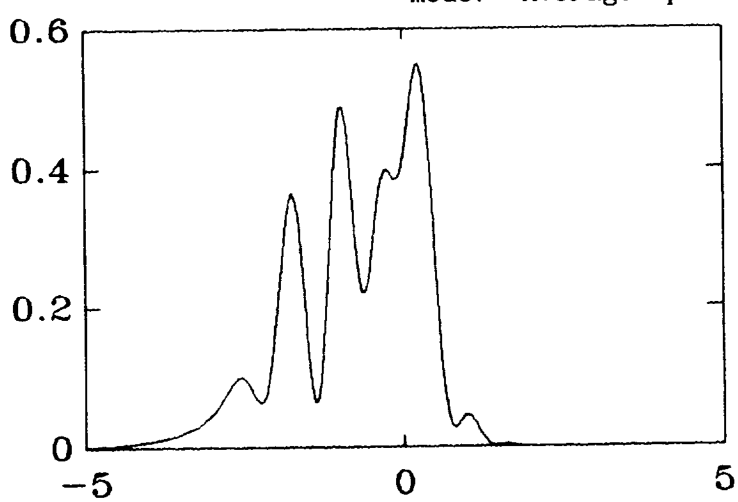

Fig. 4 Quantum interference in $P(x) = \langle x|\rho|x\rangle$ for the fundamental mode at $\tau = 4$. Average photon number N = 10.